Finding the Dynamics of an Integrable Quantum Many-Body System via Machine Learning

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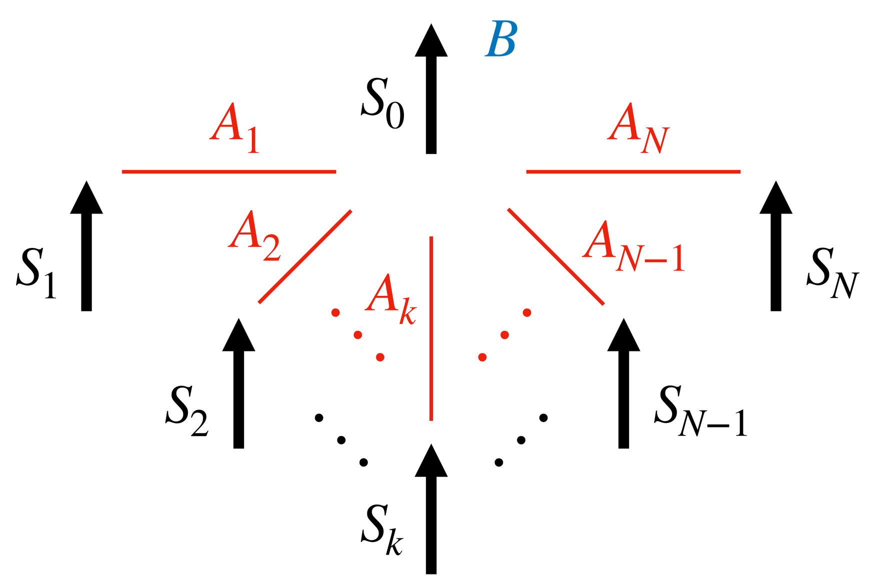
1. Model description

- Our system of interest is a central-spin model (Gaudin magnet) consisting of one central spin coupled to N environmental spins.



- The Hamiltonian is given by

$$H = BS_0^z + \sum_{k=1}^{N} A_k \mathbf{S}_0 \cdot \mathbf{S}_k$$

$$A_k = \frac{A}{N_0} e^{\frac{-(k-1)}{N_0}} \text{ and } N_0 = (N+1)/2$$

2. Variational algorithms

- We use the restricted Boltzmann machine (RBM) as our variational ansatz, where the unnormalized amplitude of a particular spin configuration in the computational basis is given by

$$\Psi_{\mathcal{W}}(\sigma) = \sum_{\{h_i\}} e^{\sum_j a_j \sigma_j + \sum_i b_i h_i + \sum_{ij} w_{ij} h_i \sigma_j}$$

- To find the ground state, we need to minimize the energy. The estimated gradient is

$$F_i = \frac{\partial E}{\partial \mathcal{W}_i^*} = \langle E_{\text{local}} \mathcal{O}_i^\dagger \rangle_{\tilde{\sigma}} - \langle E_{\text{local}} \rangle_{\tilde{\sigma}} \langle \mathcal{O}_i^\dagger \rangle_{\tilde{\sigma}}$$

where the variational logarithmic derivative of the RBM state vector is defined as

$$\mathcal{O}_k = \frac{1}{\Psi_{\mathcal{W}}(\sigma)} \partial_{\mathcal{W}_k} \Psi_{\mathcal{W}}(\sigma)$$

3. Excited states

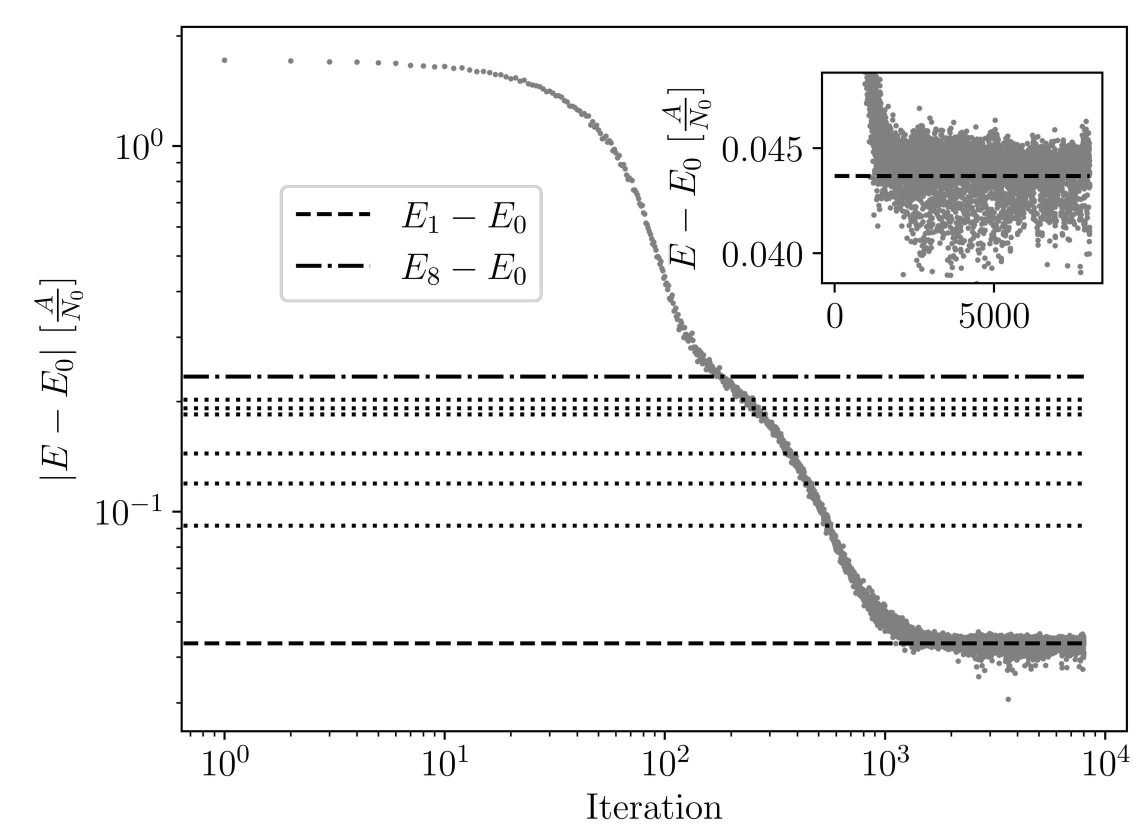
- The n[th] excited state is the ground state of the Hamiltonian

$$H_n = H + \sum_{j=0}^{n-1} \beta_j \frac{|\mathcal{W}^j\rangle\langle\mathcal{W}^j|}{\langle\mathcal{W}^j|\mathcal{W}^j\rangle}$$

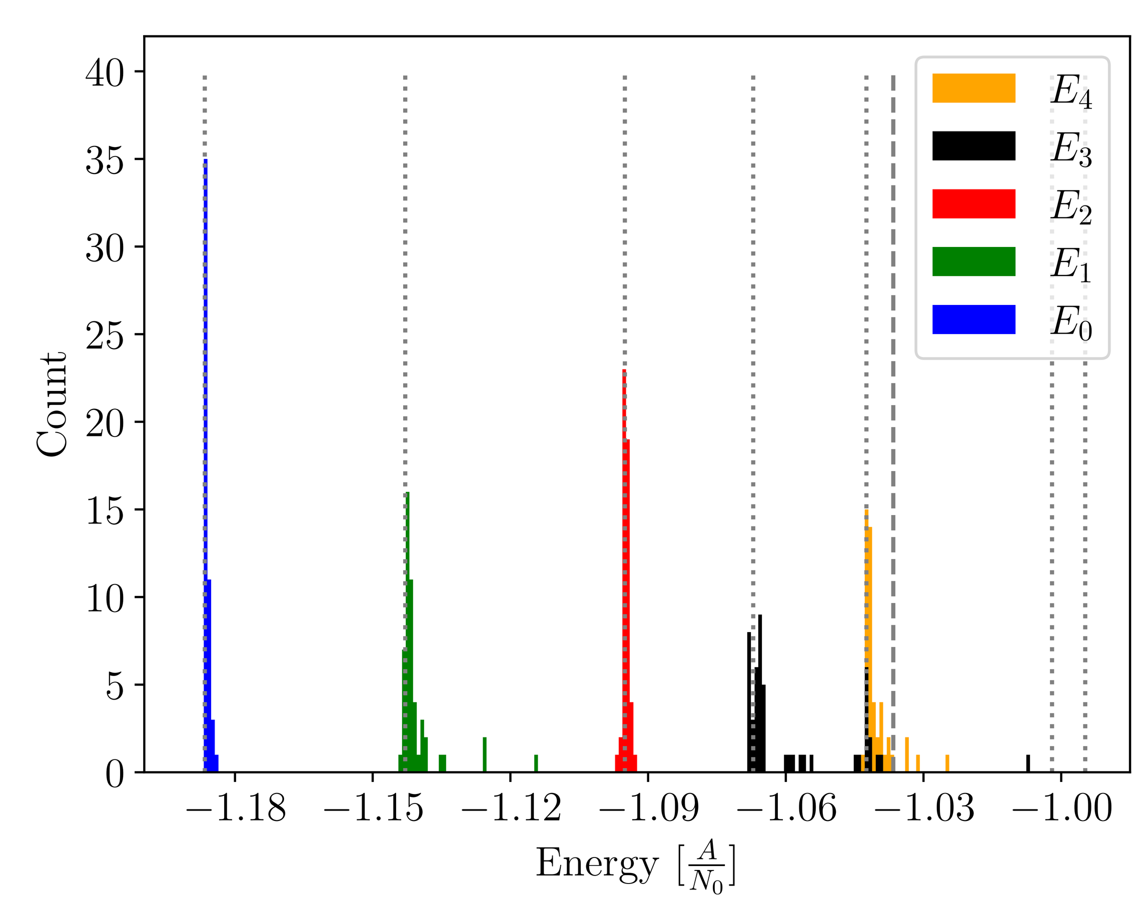
- We now need to minimize the energy plus a set of penalty terms. The gradient then becomes

$$\tilde{F}_i^n = \frac{\partial \tilde{E}_n}{\partial \mathcal{W}_i^*} = \langle E_{\text{local}} \mathcal{O}_i^\dagger \rangle_{\tilde{\sigma}} - \langle E_{\text{local}} \rangle_{\tilde{\sigma}} \langle \mathcal{O}_i^\dagger \rangle_{\tilde{\sigma}}$$
$$+ \sum_{j=0}^{n-1} \beta_j \left\langle \frac{\Psi_{\mathcal{W}^j}}{\Psi_{\mathcal{W}}} - \left\langle \frac{\mathcal{W}^j}{\Psi_{\mathcal{W}}} \right\rangle_{\tilde{\sigma}} \right\rangle_{\tilde{\sigma}} \langle \mathcal{O}_i^\dagger \rangle_{\tilde{\sigma}} \left\langle \frac{\Psi_{\mathcal{W}}}{\Psi_{\mathcal{W}^j}} \right\rangle_{\tilde{\sigma}_j}$$

- A typical successful run resulting in the first excited state of the central spin model with N=5

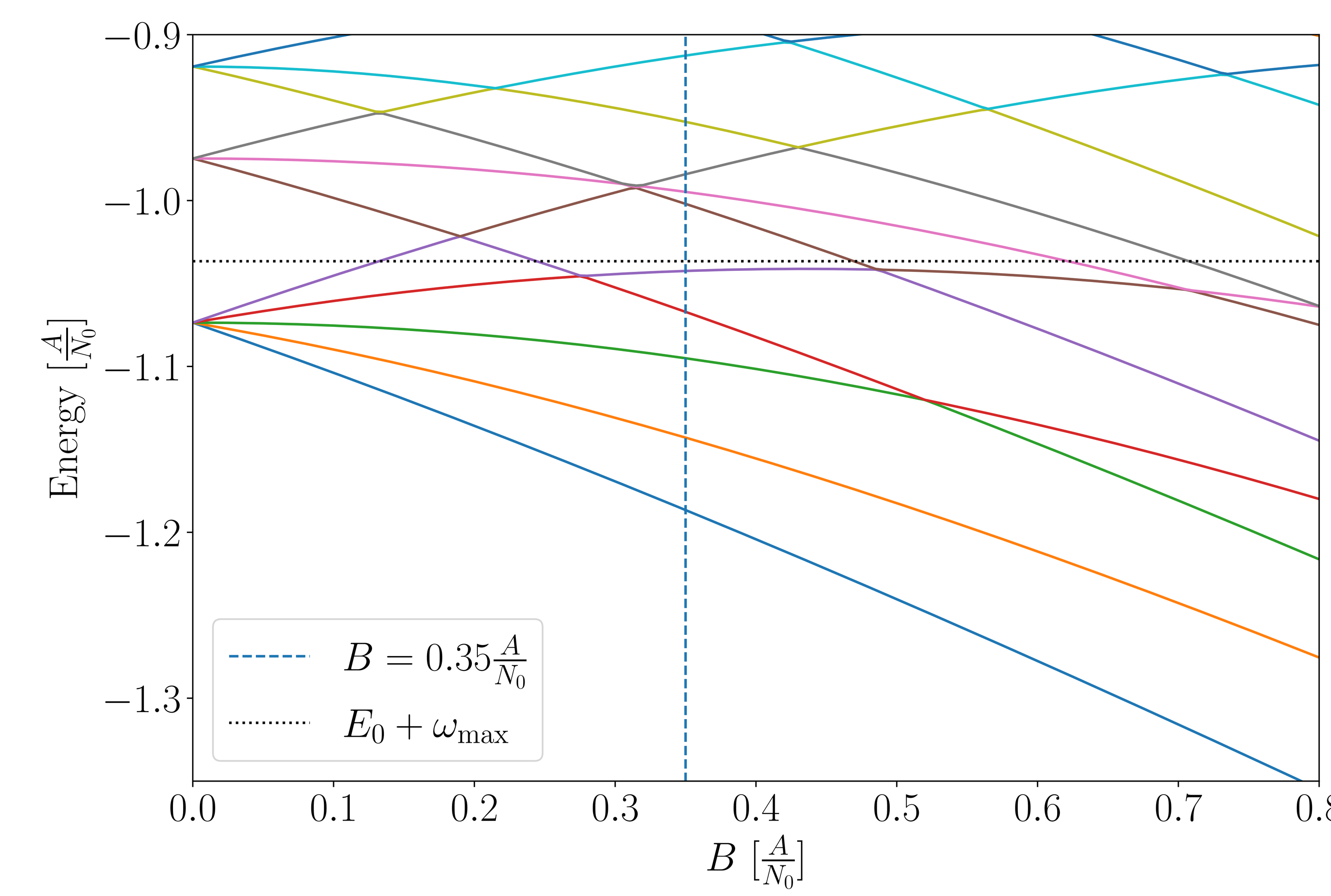


- Histogram of approximate eigenstate energy over 50 runs



4. Dynamical response

- We choose a cutoff frequency $\omega_{\max}$, where we only calculate eigenstates of energy up to $E_0 + \omega_{\max}$.



- With an accurate representation for both the low-lying energy spectrum and eigenstates, we can find the linear response of an observable under perturbation.

$$H_{total} = H + V(t) = H + f(t) \cdot S_0^y$$

$$\chi^{(xy)}(t) = -i\theta(t) \langle [S_0^x(t), S_0^y] \rangle_0$$

$$\langle S_0^x(t) \rangle = i \int_0^t dt' f(t - t') \cdot \chi^{(xy)}(t')$$